

# Multispin magnons on deformed $AdS_3 \times S^3$

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## Abstract

In this paper, we study multispin *giant* magnon configurations over deformed  $AdS_3 \times S^3$  background. In the first part of the paper, we explore two spin giant magnon bound states considering the conformal gauge conditions associated with the Polyakov action. We compute all the conserved quantities associated with the stringy dynamics in the limit of the large t'Hooft coupling ( $\lambda \gg 1$ ) where we explicitly solve the world-sheet fluctuations as well as the Virasoro constraints over the deformed geometry. In the second part of our analysis, we perform similar analysis corresponding to three spin giant magnon configurations. In both the examples, we observe the emergence of non trivial background deformations in the giant magnon dispersion relations. In our analysis, we *analytically* compute the effects associated with these background deformations ( $\kappa$ ) which vanishes in the limit,  $\kappa \rightarrow 0$ .

## 1 Overview and Motivation

According to the celebrated  $AdS_5/CFT_4$  correspondence [1], the type IIB string theory formulated in  $AdS_5 \times S^5$  background is dual to strongly coupled  $\mathcal{N} = 4$  SYM in four dimensions. Given this astonishing prescription, one could in fact think of several remarkable implications and/or consequences that naturally emerges out of this duality conjecture. One immediate consequence of this duality conjecture turns out to be the obvious equivalence between the spectrum of stringy excitation in  $AdS_5 \times S^5$  to that with the spectrum of operator dimensions in  $\mathcal{N} = 4$  SYM theory. In order to test this duality conjecture, one therefore needs to check the full quantum spectrum on both sides of the duality which is undoubtedly a difficult job in itself. However, it turns out that this situations seems to get quite manageable under certain special circumstances namely, in the limit where the number of colors becomes large,  $N \gg 1$ . This is the so called planar limit of the duality where one could in principle carry out semi-classical computations on the string theory side in order to compare it with the spectrum of anomalous dimensions corresponding to single trace (gauge invariant) operators on the dual gauge theory side. In other words, the theory becomes *integrable* on both sides of the duality [2].

A remarkable breakthrough along this direction came through the proposal due to Minahan and Zarembo [3] who sort of unveiled an astonishing connection between spin

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chains and that of the stringy dynamics in  $AdS_5 \times S^5$  by identifying the Hamiltonian operator corresponding to the spin chain systems to that with the dilatation operator in  $\mathcal{N} = 4$  SYM. Building on these ideas, several proposals came up and various attempts were made [4]-[16] in order to understand this connection to some deeper extent.

One remarkable achievement along this particular direction came through the discovery of the underlying connection between the physics of the spin wave (magnon like) excitation associated with long spin chains to that with certain specific (rotating and pulsating) stringy configurations in  $AdS_5 \times S^5$  [17]-[40]. In the following we elaborate on this issue in a bit detail. We consider the limit where one of the conserved charges ( $J$ ) of the dual  $SO(6)$  symmetry becomes infinitely large. This is the so called limit where one considers infinitely long chain of single trace operators on one side of the duality and infinitely long strings on the other side that eventually simplifies the computation enormously. On the dual gauge theory side one considers operators with large scaling dimensions namely,  $\Delta \geq J$  such that the difference,  $\Delta - J$  always appears to be finite while keeping the t'Hooft coupling ( $\lambda$ ) of the theory fixed. On the dual stringy picture one recovers identical picture where both the energy of excitation ( $E$ ) and the angular momentum ( $J$ ) of the string becomes infinitely large while keeping the difference finite.

In order to understand the physics of spin waves associated with spin chain systems more rigorously, one could imagine long single trace operators on the dual gauge theory side which has  $J$  number of operator ( $Z$ ) insertion in it. This  $Z$  essentially stands for the ground state of the spin chain configuration. In order to add excitation to this spin chain configuration one could imagine adding an operator ( $Y$ ) from outside and consider all possible operator insertion namely,

$$\mathcal{O} \sim \sum_k e^{ikp} (ZZ....ZY Z...ZZZ) \quad (1)$$

which eventually corresponds to the propagation of the *magnon* excitation over this infinitely long spin chain.

Using SUSY, it was Beisert [41] who first computed the spectrum associated with the above spin chain system and arrived at the following dispersion relation corresponding to these magnon excitation associated with the spin chain configuration,

$$\Delta - J = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} \quad (2)$$

which in the limit of the large t'Hooft coupling ( $\lambda \gg 1$ ) further simplifies to,

$$\Delta - J = \frac{\sqrt{\lambda}}{\pi} \left| \sin \frac{p}{2} \right| \quad (3)$$

where,  $p$  is the momentum associated with the single magnon excitation.

In the dual string theory description, the magnon dispersion relation (3) was recovered by Hofman and Maldacena [17] by considering open strings in  $R \times S^2$  which is a subspace of the full  $AdS_5 \times S^5$  geometry. In the dual string theory description, these magnon excitation could be thought of as being localized solitonic excitation propagating over an infinitely long string moving in  $R \times S^2$ . In their analysis [17], Hofman and Maldacena considered open strings rotating on the equator of  $S^2$  that maintains a constant angular separation ( $\Delta\varphi$ ) between its two endpoints which they finally identify as a geometric realization of the magnon momentum ( $p$ ) associated with the spin chain system.

It turns out that apart from having the elementary magnon excitation, the asymptotic spectrum associated with the spin chain system also contains bound states of magnon excitation which therefore motivates one to go one step further and generalize the earlier observations [17] for multispin magnon excitation, in particular for two spin magnon bound states [42]-[48] for which the dispersion relation takes the following form,

$$E - J_1 = \sqrt{J_2^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} \quad (4)$$

where,  $J_2$  is the second spin of the string which corresponds to the number of magnons in a particular bound state characterized by its energy of excitation ( $E$ ) and the large spin quantum number  $J_1$ . In other words, from the point of view of (1), one could think of two spin magnon excitation as being the bound state of  $J_2$  number of  $Y$  excitation. Clearly, for  $J_2 = 1$  one recovers the elementary magnon excitation. In principle, the relation (4) is valid for all values of  $J_2$  and  $\lambda$ . However, in the present analysis, we shall be concerned with the limit,  $\lambda \gg 1$ ,  $J_1 \rightarrow \infty$  and  $J_2 \sim \sqrt{\lambda}$ . The most remarkable fact about both the dispersion relations (3) and (4) is that they exhibit a finite difference between two diverging entities ( $E \sim \Delta$  and  $J$ ) of the theory even at finite t'Hooft coupling ( $\lambda$ ).

The results corresponding to two spin giant magnon configurations had been subsequently generalized for the three spin case [21],[49] where considering the dressing techniques [21], the three spin giant magnon solutions with three conserved charges  $J_1(\rightarrow \infty)$ ,  $J_2$  and  $\Psi$  were constructed. These solutions could be thought of as being the superposition of two two spin non interacting bound states of magnons with equal and opposite momenta. At this stage, it is noteworthy to mention that there exists a completely equivalent as well as parallel way of looking at magnon excitation in terms of classical sine-Gordon theory which enable us to look at giant magnon solutions as being that of the solitonic solutions within these special classes of integrable models [44]. In fact, this issue had been systematically addressed by constructing the two spin magnon excitation as solitonic solutions within the framework of Complex sine-Gordon theory [44].

In order to understand the goal of the present analysis, it is first customary to note that all the above discussions regarding the multispin magnon bound states are solely based on the integrable structure of superstring theories in  $AdS_5 \times S^5$  background without any deformations. However, very recently there have been numerous attempts to extend this vision beyond the usual notion of  $AdS_5 \times S^5$  and in particular to explore the issue of integrability over deformed geometries namely, by constructing one parameter integrable deformations [50]-[71] of  $AdS_5 \times S^5$  those are not related to T duality transformations.

Explorations regarding the integrable structure of superstring theories over these deformed geometries turn out to be an absolutely essential question to be addressed from various points of view. The first and the foremost issue is related to the question of finding out the dual gauge theory description associated with these deformed geometries. The answer to this question should be certainly non trivial and in fact is quite involved. One of the obvious reasons behind this non triviality lies over the fact that the original  $SO(2,4) \times SO(6)$  symmetry associated with the background geometry is now broken down to its subset. The logical step in this connection would therefore be to explore the dispersion relations associated with the multispin magnon bound states over these deformed geometries by considering their low dimensional analogues. A systematic analysis of which is still lacking in the literature and the purpose of the present paper is therefore to fill up this gap at the earliest.

In the present analysis, we compute multispin magnon dispersion relations over the  $\kappa$ - deformed  $AdS_3 \times S^3$  background [51] which is essentially a  $3d$  truncated version of the original metric corresponding to the deformed  $AdS_5 \times S^5$  model [51]. The most notable feature of this reduced model is that the  $B$  field of the original  $AdS_5 \times S^5$  model simply vanishes during this  $3d$  reduction procedure [51]. The deformed  $AdS_3 \times S^3$  is special as well as interesting from the point of view of its interpolating structure between the pure  $AdS_3 \times S^3$  near the limit,  $\kappa \rightarrow 0$  and that of the  $dS_3 \times H^3$  in the limit,  $\kappa \rightarrow \infty$  [51].

We start our analysis by *analytically* constructing the spin two giant magnon configurations over the  $\kappa$ - deformed  $AdS_3 \times S^3$  background in Section 2. We explore the dynamics emerging out of the Polyakov action for open strings over this deformed geometry in the limit of the large t'Hooft coupling ( $\lambda \gg 1$ ) where we compute the the conserved quantities associated with the stringy dynamics by exploring the corresponding equations of motion as well as the Virasoro constraints.

In Section 3, we compute the energy of excitation ( $E \sim \Delta$ ) as well as the two angular momenta,  $J_1(\rightarrow \infty)$  and  $J_2$  associated with the stringy dynamics whose combination seem to satisfy a spin two *giant* magnon like dispersion relation [44] together with some non trivial contributions sourced due the  $\kappa$ - deformations of the background geometry. Our analysis is quite suggestive to speculate the following dispersion relation associated with the mysterious dual gauge theory description that consists of spin two giant magnon bound states,

$$\Delta - J_1 - \sqrt{J_2^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} = \mathcal{F}(\kappa^2) \quad (5)$$

where, the R.H.S. is a non trivial function of the deformation parameter ( $\kappa$ ) which however vanishes smoothly in the limit,  $\kappa \rightarrow 0$ . In our analysis, using the dual (*classical*) string theory picture, we perform an explicit analytic computation in order to determine this function  $\mathcal{F}(\kappa^2)$  finally upto quadratic order in the deformation parameter ( $\kappa$ ). However, in our analysis, we provide sufficient ingredients namely, the *exact* analytic expressions for the conserved entities (in particular for the two spin configurations) in the deformation parameter ( $\kappa$ ) so that one could easily calculate the exact form of the function  $\mathcal{F}(\kappa^2)$ .

In order to complete our discussions on multispin giant magnon bound states, we perform a further analysis corresponding to spin three giant magnon excitation in Section 4 where we carry out almost identical computations similar to that for the spin two case. Like in the two spin case, considering the conformal gauge, we explore the equations of motion as well as the Virasoro constraints associated with the stringy dynamics over the  $\kappa$ - deformed background where we finally push ourselves towards the regime of small deformations ( $0 < \kappa < 1$ ) and correctly identify the allowed parameter space for the spin three magnon configuration [49] in the limit of the large t'Hooft coupling ( $\lambda \gg 1$ ). However, the notable difference between the spin three configuration and that of the spin two configuration turns out to be the issue associated with the choice of the static gauge condition. This is due to the fact that unlike the two spin configuration, the string time coordinate associated with the three spin configuration exhibits a non trivial dependence [49] on the world-sheet coordinates. Finally, in order to arrive at the desired dispersion relation, we regulate all the UV divergences associated with various conserved entities of the theory. Like in the two spin case, the final dispersion relation corresponding to the three spin (giant) configuration also exhibits non trivial  $\kappa$  dependence which we estimate analytically upto quadratic order in the deformation parameter ( $\kappa$ ).

Finally, we conclude in Section 5.

## 2 Two spin magnons: Preliminaries

We start our analysis by considering the dynamics of open strings on deformed  $AdS_3 \times S^3$  backgrounds that has been recently initiated by the authors in [50]-[51] and then subsequently explored in many other directions [57]-[58],[61]-[63],[68]. The deformed  $AdS_3 \times S^3$  background could be formally expressed as [51],

$$\begin{aligned} ds^2 &= ds_{AdS_3}^2 + ds_{S^3}^2 \\ ds_{AdS_3}^2 &= -\mathfrak{h}(\varrho)dt^2 + \mathfrak{f}(\varrho)d\varrho^2 + \varrho^2 d\psi^2 \\ ds_{S^3}^2 &= \tilde{\mathfrak{h}}(r)d\varphi^2 + \tilde{\mathfrak{f}}(r)dr^2 + r^2 d\phi^2 \end{aligned} \quad (6)$$

where, the metric coefficients turn out to be,

$$\begin{aligned} \mathfrak{h} &= \frac{1 + \varrho^2}{1 - \kappa^2 \varrho^2}, \quad \mathfrak{f} = \frac{1}{(1 + \varrho^2)(1 - \kappa^2 \varrho^2)} \\ \tilde{\mathfrak{h}} &= \frac{1 - r^2}{1 + \kappa^2 r^2}, \quad \tilde{\mathfrak{f}} = \frac{1}{(1 - r^2)(1 + \kappa^2 r^2)} \end{aligned} \quad (7)$$

such that the NS-NS two form vanishes.

Our goal would be to explore the dispersion relation corresponding to two spin giant magnons on the above background (6) in the limit of large t'Hooft coupling where one of the spin takes large value. We consider two spins of the magnon excitation to be in  $S^3$ .

We start our analysis with the Polyakov action for the open string on the deformed background (6),

$$S = -\frac{T}{2} \int_{-\pi}^{\pi} d\sigma d\tau \sqrt{-\gamma} \gamma^{\alpha\beta} \mathfrak{g}_{ab}(X) \partial_{\alpha} X^a \partial_{\beta} X^b \quad (8)$$

where, the effective string tension could be formally expressed as [51],

$$T = \frac{\sqrt{\lambda}}{2\pi} \sqrt{1 + \kappa^2}. \quad (9)$$

Here,  $\gamma_{\alpha\beta}$  is the induced metric on the world sheet and  $X^a$ s are the coordinates of the target space. Moreover, here  $\mathfrak{g}_{ab}(X)$  is the metric of the target space.

The central role of our present discussion is played by the so called Virasoro constraints which are expressed in terms of the components of the stress tensor,

$$T_{\alpha\beta} = \mathfrak{g}_{ab} \partial_{\alpha} X^a \partial_{\beta} X^b - \frac{1}{2} \gamma_{\alpha\beta} \gamma^{\mu\nu} \partial_{\mu} X^a \partial_{\nu} X^b \mathfrak{g}_{ab} = 0. \quad (10)$$

Before we actually proceed further, it is customary to note that the background (6) is invariant under the following translations namely,

$$\delta X^k = a^k, \quad k = t, \psi, \varphi, \phi \quad (11)$$

which therefore suggests that these symmetries could also be realized in the Polyakov action (8). As a result of this, it is indeed quite instructive to write down the conserved charges in the following form,

$$\mathfrak{P}_k = T \int_{-\pi}^{\pi} d\sigma \sqrt{-\gamma} \gamma^{\alpha\tau} \mathfrak{g}_{ak} \partial_{\alpha} X^a \quad (12)$$

where, corresponding to each of these  $k$ 's one should be able to recover different conserved charges present in the system. For example, the choice,  $k = t$  should give us the energy ( $E$ ) of the stringy configuration.

In order to proceed further, we choose the following ansatz,

$$t = \xi\tau, \quad \varrho = \varrho(\tau), \quad \psi = \zeta\tau, \quad r = r(\sigma, \tau), \quad \varphi = \varphi(\sigma, \tau), \quad \phi = \phi(\sigma, \tau) \quad (13)$$

along with the conformal gauge conditions namely,  $-\gamma^{\tau\tau} = \gamma^{\sigma\sigma} = 1$  and  $\gamma^{\tau\sigma} = 0$  where,  $\xi$  and  $\zeta$  are two constants.

With the above choice (13) in hand, we first go for some consistency checks. Consider the equation corresponding to,  $X^t = t$  which yields,

$$\partial_{\tau} \varrho = 0. \quad (14)$$

This clearly suggests that,  $\varrho = \varrho_0$  is basically a constant in  $\tau$ . In order to fix this constant, we compute the conserved charge corresponding to time translation namely,

$$\mathfrak{P}_t = -E = -2\pi T \xi \mathfrak{h}(\varrho_0) \quad (15)$$

which finally yields,

$$\mathfrak{h}(\varrho_0) = \frac{E}{2\pi \xi T}. \quad (16)$$

On the other hand, the equation corresponding to,  $X^{\varrho} = \varrho$  yields,

$$\frac{\xi^2 \varrho_0 (1 + \kappa^2)}{(1 - \kappa^2 \varrho_0^2)^2} - \varrho_0 \zeta^2 = 0. \quad (17)$$

Now, this yields two possibilities. One is the possibility that,  $\varrho_0 = 0$ . The other possibility comes through the combination of (16) and (17) which finally yields,

$$\varrho_0 = \left( \frac{E}{2\pi T \zeta} \sqrt{1 + \kappa^2} - 1 \right)^{1/2}. \quad (18)$$

This further puts constraints on the energy namely,

$$E \geq \frac{2\pi T \zeta}{\sqrt{1 + \kappa^2}}. \quad (19)$$

Finally, we note that following our construction the equation corresponding to  $\psi$  is trivially satisfied. This further yields the corresponding conserved charge as,

$$\mathfrak{P}_{\psi} = 2\pi T \zeta \varrho_0^2. \quad (20)$$

In our analysis, however, we consider the first possibility namely,  $\varrho_0 = 0$  which yields,  $\mathfrak{h}(\varrho_0) = 1$  and,  $\mathfrak{P}_\psi = 0$ . With this choice, the effective background geometry seen by the string eventually reduces to,  $R \times S^3$  which is a subspace of the full deformed geometry.

Our next task would be to explore the dynamics of the rest of the three other basic variables. In order to do that, we closely follow the method developed in [18],[57] where we consider the Virasoro constraints (10) rather than considering the equations of motion directly. It turns out that these Virasoro constraints for the present system yield,

$$\begin{aligned} T_{\sigma\sigma} &= -\frac{E^2}{4\pi^2 T^2} + \tilde{\mathfrak{h}}(r)((\partial_\sigma \varphi)^2 + (\partial_\tau \varphi)^2) + \tilde{\mathfrak{f}}(r)((\partial_\sigma r)^2 + (\partial_\tau r)^2) \\ &\quad + r^2((\partial_\sigma \phi)^2 + (\partial_\tau \phi)^2) = 0 = T_{\tau\tau} \\ T_{\tau\sigma} &= \tilde{\mathfrak{h}}(r)\partial_\sigma \varphi \partial_\tau \varphi + \tilde{\mathfrak{f}}(r)\partial_\sigma r \partial_\tau r + r^2\partial_\sigma \phi \partial_\tau \phi = 0. \end{aligned} \quad (21)$$

The natural next task would be solve these Virasoro constraints. In order to solve these constraints, we choose the following ansatz,

$$\varphi = \omega\tau + \mathfrak{s}(\varsigma), \quad \phi = \tau + \mathfrak{q}(\varsigma), \quad r = r(\varsigma) \quad (22)$$

where, the new variable,  $\varsigma = \sigma - v\omega\tau$  is the linear combination of the world sheet coordinates  $(\sigma, \tau)$ . Substituting (22) back into (21) we obtain the following set of constraint equations,

$$\begin{aligned} -\frac{E^2}{4\pi^2 T^2} + \tilde{\mathfrak{h}}(r)(\mathfrak{s}'^2 + (\omega - v\omega\mathfrak{s}')^2) + \tilde{\mathfrak{f}}(r)r'^2(1 + (v\omega)^2) + r^2(\mathfrak{q}'^2 + (1 - v\omega\mathfrak{q}')^2) &= 0 \\ \tilde{\mathfrak{h}}(r)\mathfrak{s}'\omega(1 - v\mathfrak{s}') - v\omega\tilde{\mathfrak{f}}(r)r'^2 + r^2\mathfrak{q}'(1 - v\omega\mathfrak{q}') &= 0 \end{aligned} \quad (23)$$

where, the prime indicates derivative w.r.t the variable  $\varsigma$ . Clearly, for two spin magnons we encounter a different situation where one needs to solve for three different variables instead of two. This is due to obvious reasons as we have included an additional conserved quantity (charge) into our theory namely, the second angular momentum ( $J_2$ ).

After some trivial algebra we note,

$$\begin{aligned} \mathfrak{s}' &= \frac{v}{\tilde{\mathfrak{h}}(r)} \frac{(\xi^2 - \omega^2\tilde{\mathfrak{h}}(r))}{(1 - v^2\omega^2)} - \frac{r^2v}{\tilde{\mathfrak{h}}(r)} \frac{(1 + \frac{\mathfrak{q}'}{\omega v}(1 - v^2\omega^2))}{(1 - v^2\omega^2)} \\ r'^2 &= \frac{\Xi(r)}{\tilde{\mathfrak{f}}(r)\tilde{\mathfrak{h}}(r)(1 - \omega^2v^2)^2} + \frac{r^2\mathfrak{q}'}{\tilde{\mathfrak{f}}(r)v\omega}(1 - v\omega\mathfrak{q}') \end{aligned} \quad (24)$$

where, the function,  $\Xi(r)$  could be formally expressed as,

$$\Xi(r) = (\xi^2 - \omega^2\tilde{\mathfrak{h}}(r) - r^2(1 + \frac{\mathfrak{q}'}{v\omega}(1 - v^2\omega^2)))(\tilde{\mathfrak{h}}(r) - v^2\xi^2 + v^2r^2(1 + \frac{\mathfrak{q}'}{v\omega}(1 - v^2\omega^2))). \quad (25)$$

Our next task would be substitute  $\mathfrak{q}'$  using its E.O.M. where, we consider all the fluctuations on the string world sheet to be sufficiently small as well as restrict ourselves corresponding to the large values of the t'Hooft coupling ( $\lambda \gg 1$ ). In this limit, the characteristic radius of curvature associated with the target space becomes sufficiently large compared to that of the string length scale. As a result, all the subleading corrections associated with the ( $\alpha'$  expansion) derivatives of the metric ( $\mathfrak{g}_{ab}(X)$ ) are highly

suppressed compared to that of the leading term in the Polyakov action (8). Following these systematic steps, one finally obtains,

$$\begin{aligned} \mathfrak{s}' &\approx \frac{v}{\tilde{\mathfrak{h}}(r)} \frac{(\xi^2 - \omega^2 \tilde{\mathfrak{h}}(r))}{(1 - v^2 \omega^2)} \\ r'^2 &\approx \frac{(\xi^2 - \omega^2 \tilde{\mathfrak{h}}(r))(\tilde{\mathfrak{h}}(r) - v^2 \xi^2) - r^2 \tilde{\mathfrak{h}}(r)}{\tilde{\mathfrak{f}}(r) \tilde{\mathfrak{h}}(r) (1 - \omega^2 v^2)^2} = - \frac{(r^2 - r_{min}^2)(r_{max}^2 - r^2)}{(1 - r^2) \tilde{\mathfrak{f}}(r) (1 - \omega^2 v^2)^2}. \end{aligned} \quad (26)$$

where,  $r_{max}$  and  $r_{min}$  correspond to the two extremum values for which the function  $r'$  vanishes. These for the present case turn out to be,

$$r_{min,max} = \sqrt{\frac{1 - \alpha(\beta^2 + \gamma^2)}{2|1 - \alpha|}} \left( 1 \pm \sqrt{1 + \frac{4\beta^2 \gamma^2 \alpha |1 - \alpha|}{(1 - \alpha(\beta^2 + \gamma^2))^2}} \right)^{1/2} \quad (27)$$

where,  $\pm$  correspond to the minimum and the maximum values respectively. Furthermore, different parameters appearing in (27) could be formally expressed as,

$$\beta = \sqrt{\frac{\omega^2 - \xi^2}{\omega^2 + \kappa^2 \xi^2}}, \quad \gamma = \sqrt{\frac{1 - v^2 \xi^2}{1 + \kappa^2 v^2 \xi^2}}, \quad \alpha = (\omega^2 + \kappa^2 \xi^2)(1 + \kappa^2 v^2 \xi^2). \quad (28)$$

### 3 Conserved charges

With the above machinery in hand, we now proceed towards computing various conserved charges associated with the stringy dynamics in the bulk. We first compute the conserved charged associated with the angular coordinate  $\varphi$  namely,

$$\mathfrak{P}_\varphi = J_1 = 2T \int_{r_{min}}^{r_{max}} \frac{\omega}{|r'|} \tilde{\mathfrak{h}}(r) (v \mathfrak{s}' - 1) dr. \quad (29)$$

Using (26), this further yields,

$$J_1 = 2\omega T (1 + \kappa^2 v^2 \xi^2) \int_{r_{min}}^{r_{max}} \frac{(r^2 - r_0^2)}{(1 + \kappa^2 r^2)^{3/2}} \frac{dr}{\sqrt{(r^2 - r_{min}^2)(r_{max}^2 - r^2)}} \quad (30)$$

where,  $r_0^2 = \frac{1 - v^2 \xi^2}{1 + \kappa^2 v^2 \xi^2}$ . Our next task would be to perform the above integral (30) and identify the limit where it diverges. We first consider the limit,  $r_{min} \rightarrow 0$ . Performing the above integral we note,

$$J_1 = 2\omega T \mathcal{I}(r) \Big|_{r=0}^{r=r_{max}} \quad (31)$$

where, the *exact* analytic form of the function  $\mathcal{I}(r)$  could be formally expressed as,

$$\mathcal{I}(r) = \frac{(\kappa^2 \xi^2 v^2 + 1) \mathcal{N}(r)}{\mathcal{D}(r)} \quad (32)$$

where we note,

$$\begin{aligned} \mathcal{N}(r) = r_0^2 r (\kappa^2 r_{max}^2 + 1) \sqrt{1 - \frac{r_{max}^2}{r^2}} \sqrt{\frac{1}{\kappa^2 r^2} + 1} {}_2F_1 \left( 1; \frac{1}{2}, \frac{1}{2}; 2; \frac{r_{max}^2}{r^2}, -\frac{1}{r^2 \kappa^2} \right) \\ + 2r (\kappa^2 r_0^2 + 1) (r^2 - r_{max}^2) \end{aligned} \quad (33)$$



$$\mathcal{D}(r) = 2 (\kappa^2 r_{max}^2 + 1) \sqrt{r^2 (r_{max}^2 - r^2)} \sqrt{\kappa^2 r^2 + 1}. \quad (34)$$

However, since in our analysis, we shall be finally interested in the small  $\kappa$  regime in order to match our results to that with the known dispersion relations without any background deformations, therefore we finally expand the function,  $\mathcal{I}(r)$  perturbatively in  $\kappa$  which yields the following,

$$\begin{aligned} \mathcal{I}(r) = & F_1 \left( 1; \frac{1}{2}, \frac{1}{2}; 2; \frac{r_{max}^2}{r^2}, -\frac{1}{\kappa^2 r^2} \right) \left( \frac{r_0^2}{2\kappa r^2} \right) (1 + \kappa^2 \xi^2 v^2) \\ & - \frac{\sqrt{r_{max}^2 - r^2}}{2} (2 - \kappa^2 (-2\xi^2 v^2 - 2r_0^2 + 2r_{max}^2 + r^2)) + \mathcal{O}(\kappa^3) \end{aligned} \quad (35)$$

where,  $F_1$  is the so called Appell function of the first kind. Clearly, the above integral (31) possesses a divergent piece near the limit,  $r \rightarrow 0$ . This essentially corresponds to the large spin ( $J$ ) limit of the giant magnon solution [17],[57]. Henceforth, we perform our analysis in this large spin limit. We separate out the divergent piece from the finite piece which finally yields,

$$\begin{aligned} J_1 = & 2F_1 \left( 1; \frac{1}{2}, \frac{1}{2}; 2; 1, -\frac{1}{\kappa^2 r_{max}^2} \right) \left( \frac{\omega T r_0^2}{2\kappa r_{max}^2} \right) (1 + \kappa^2 \xi^2 v^2) \\ & + 2\omega T r_{max} (1 + \kappa^2 (\xi^2 v^2 + r_0^2 - r_{max}^2)) - 2\omega T J_1^{(\epsilon)} \end{aligned} \quad (36)$$

where,  $|\epsilon| \ll 1$  is some cutoff such that the entity,

$$J_1^{(\epsilon)} = F_1 \left( 1; \frac{1}{2}, \frac{1}{2}; 2; \frac{r_{max}^2}{\epsilon^2}, -\frac{1}{\kappa^2 \epsilon^2} \right) \left( \frac{(1 - \xi^2 v^2)}{2\kappa \epsilon^2} \right) \quad (37)$$

correspond to the divergent contributions to the angular momentum in the large  $\lambda$  limit.

Next, we compute the energy ( $E = 2\pi\xi T$ ) of the stringy configuration corresponding to this large  $J_1$  limit. In order to do that, we first note,

$$\begin{aligned} 2\pi &= \int_{-\pi}^{\pi} d\sigma = 2 \int_0^{r_{max}} \frac{dr}{|r'|} \\ &= \frac{r \sqrt{1 - \frac{r_{max}^2}{r^2}} (v^2 \omega^2 - 1) \sqrt{\frac{1}{\kappa^2 r^2} + 1} F_1 \left( 1; \frac{1}{2}, \frac{1}{2}; 2; \frac{r_{max}^2}{r^2}, -\frac{1}{r^2 \kappa^2} \right)}{2 \sqrt{r^2 (r_{max}^2 - r^2)} (r_{max}^2 + r) \sqrt{\kappa^2 r^2 + 1}} \end{aligned} \quad (38)$$

which is again an *exact* expression in the deformation parameter,  $\kappa$ . However, following our previous arguments, we expand the above expression (38) perturbatively in the deformation parameter ( $\kappa$ ) which finally yields,

$$2\pi \approx -2 \left( \frac{(1 - \omega^2 v^2)}{2\kappa r_{max}^2} \right) F_1 \left( 1; \frac{1}{2}, \frac{1}{2}; 2; 1, -\frac{1}{\kappa^2 r_{max}^2} \right) + E^{(div)} \quad (39)$$

where,

$$E^{(div)} = 2F_1 \left( 1; \frac{1}{2}, \frac{1}{2}; 2; \frac{r_{max}^2}{\epsilon^2}, -\frac{1}{\kappa^2 \epsilon^2} \right) \left( \frac{(1 - \omega^2 v^2)}{2\kappa \epsilon^2} \right) \quad (40)$$

possesses the usual divergences. Finally, using (36) and (38) and setting,  $\omega = -\xi$  we get rid of these divergences and arrive at the following entity,

$$E - J_1 = 2\xi T r_{max} (1 + \kappa^2 (\xi^2 v^2 + r_0^2 - r_{max}^2)) \quad (41)$$

which is finite in spite of the fact that each of them is diverging as individuals [17].

Finally, we are left with the third and the last conserved charge associated with our system namely, the second angular momentum,  $J_2$  which for the present case turns out to be,

$$\mathfrak{P}_\phi = J_2 = \frac{2T \tan^{-1}(\kappa r_{max})}{\kappa} \approx 2T r_{max} \left( 1 - \frac{\kappa^2 r_{max}^2}{3} \right) + \mathcal{O}(\kappa^3). \quad (42)$$

Our final task would be to compute the momentum ( $p$ ) associated with these two spin magnon excitation. In AdS/CFT, the magnon momentum ( $p$ ) associated with the excitation of the spin chain system could be realized as the geometric entity. It is basically related to the angle of separation ( $\Delta\varphi$ ) between the two end points of the string namely,

$$\frac{p}{2} = \frac{\Delta\varphi}{2} = \int_0^{r_{max}} \frac{\varphi'}{|r'|} dr. \quad (43)$$

Performing the above integral (43) and rescaling the momentum we finally obtain,

$$\begin{aligned} \sin \frac{\tilde{p}}{2} &\approx r_{max} - \left( \frac{\kappa^2 \mathfrak{b}}{2\sqrt{|1 - r_{max}^2|}} \right) + \mathcal{O}(\kappa^6) \\ \mathfrak{b} &= \frac{(2\xi^2 v^2 - 1)}{\sqrt{|1 - r_{max}^2|}} \sin^{-1} r_{max} - r_{max} \end{aligned} \quad (44)$$

where, we have used the fact that,  $|r_0^2/r^2| \ll 1$ .

Finally, using (41), (42) and (44) we arrive at the following dispersion relation corresponding to the two spin giant magnon excitations in  $\kappa$ -deformed  $AdS_3 \times S^3$  in the limit of the large t'Hooft coupling,

$$E - J_1 - \sqrt{J_2^2 + \frac{\hat{\lambda}}{\pi^2} \sin^2 \frac{\tilde{p}}{2}} = \mathcal{F}(\kappa^2) = \sqrt{\lambda} \kappa^2 \left( \frac{\sqrt{2} r_{max}}{\pi} \right) \mathcal{K} + \mathcal{O}(\kappa^4) \quad (45)$$

where,  $\hat{\lambda} = \lambda(1 + \kappa^2)$  and we have set,  $\xi = \sqrt{2}$ . The entity,  $\mathcal{K}$  is basically a constant and could be formally expressed as,

$$\mathcal{K} = 2v^2 + r_0^2 - \frac{1}{2} - \frac{5}{6} r_{max}^2 + \frac{\mathfrak{b}}{4r_{max} \sqrt{|1 - r_{max}^2|}}. \quad (46)$$

Eq.(45) essentially summarizes the dispersion relation corresponding to two spin giant magnon bound states over the  $\kappa$ -deformed [51]  $AdS_3 \times S^3$  background in the infinite  $J_1$  limit. This relation clearly reduces to the known dispersion relation for giant magnon bound states corresponding to string sigma models [44] in  $R \times S^3$  in the limit of the vanishing ( $\kappa \rightarrow 0$ ) deformation. In summary, from the above analysis, it turns out that in the presence of  $\kappa$  deformations [51] the R.H.S. of the dispersion relation (45) corresponding to the two spin magnon bound states could in principle be non vanishing depending on the various parameters of the theory. However, the corresponding interpretation from the dual gauge theory point of view is not very clear at this moment and which is thereby worthy for further investigations.

## 4 Three spin magnons

As a possible extension of our previous analysis, we now study the dispersion relation corresponding to three spin giant magnons over the  $\kappa$ -deformed background (6) in the limit of the large t'Hooft ( $\lambda \gg 1$ ) coupling. We consider one of the spins  $\Psi$  in  $AdS_3$  and the other two spins,  $J_1$  and  $J_2$  along the two orthogonal directions of the three sphere ( $S^3$ ). In order to proceed further, we consider the following ansatz [49],

$$\begin{aligned} t &= \tau + \mathbf{t}(\varsigma), \quad \varrho = \varrho(\varsigma), \quad \varphi = \omega_1 \tau + \mathbf{s}(\varsigma) \\ \phi &= \tau + \mathbf{q}(\varsigma), \quad r = r(\varsigma), \quad \psi = \omega_2 \tau + \mathbf{p}(\varsigma) \end{aligned} \quad (47)$$

where,  $\varsigma = \sigma - v\tau$ .

In the following, we first enumerate all the conserved quantities those are of interest to us namely,

$$\begin{aligned} E &= T \int_{-\pi}^{\pi} d\sigma \mathfrak{h}(\varrho)(1 - v\mathbf{t}') \\ \Psi &= -T \int_{-\pi}^{\pi} d\sigma \varrho^2(\omega_2 - v\mathbf{p}') \\ J_1 &= -T \int_{-\pi}^{\pi} d\sigma \tilde{\mathfrak{h}}(r)(\omega_1 - v\mathbf{s}') \\ J_2 &= -T \int_{-\pi}^{\pi} d\sigma r^2(1 - v\mathbf{q}'). \end{aligned} \quad (48)$$

Next, we compute the Virasoro constraints for the present example that could be formally expressed as the following set of equations namely,

$$\begin{aligned} T_{\tau\tau} = T_{\sigma\sigma} &= -\mathfrak{h}(\mathbf{t}'^2 + (1 - v\mathbf{t}')^2) + \varrho'^2 f(\varrho)(1 + v^2) + \varrho^2(\mathbf{p}'^2 + (\omega_2 - v\mathbf{p}')^2) \\ &\quad + \tilde{\mathfrak{h}}(\mathbf{s}'^2 + (\omega_1 - v\mathbf{s}')^2) + r'^2 \tilde{f}(r)(1 + v^2) + r^2(\mathbf{q}'^2 + (1 - v\mathbf{q}')^2) = 0 \end{aligned} \quad (49)$$

$$\begin{aligned} T_{\tau\sigma} &= -\mathfrak{h}\mathbf{t}'(1 - v\mathbf{t}') - v\varrho'^2 f(\varrho) + \varrho^2 \mathbf{p}'(\omega_2 - v\mathbf{p}') \\ &\quad + \tilde{\mathfrak{h}}\mathbf{s}'(\omega_1 - v\mathbf{s}') - vr'^2 \tilde{f}(r) + r^2 \mathbf{q}'(1 - v\mathbf{q}') = 0. \end{aligned} \quad (50)$$

After some trivial algebra we find,

$$\begin{aligned} r'^2 &= -\frac{\mathfrak{h}\mathbf{t}'}{v\tilde{f}(r)}(1 - v\mathbf{t}') - \frac{\varrho'^2 f(\varrho)}{\tilde{f}(r)} + \frac{\varrho^2 \mathbf{p}'}{v\tilde{f}(r)}(\omega_2 - v\mathbf{p}') + \frac{\tilde{\mathfrak{h}}\mathbf{s}'}{v\tilde{f}(r)}(\omega_1 - v\mathbf{s}') + \frac{r^2 \mathbf{q}'}{v\tilde{f}(r)}(1 - v\mathbf{q}'). \\ \mathbf{s}' &= \frac{1}{\tilde{\mathfrak{h}}(r)\omega_1}(\mathfrak{h}\mathbf{t}' - \varrho^2 \omega_2 \mathbf{p}' - r^2 \mathbf{q}') + \frac{v}{\tilde{\mathfrak{h}}(r)\omega_1(1 - v^2)}(\mathfrak{h} - \tilde{\mathfrak{h}}\omega_1^2 - \varrho^2 \omega_2^2 - r^2). \end{aligned} \quad (51)$$

Our next task would be to solve for these fluctuations using their E.O.Ms (in the limit of the large t'Hooft coupling ( $\lambda \gg 1$ )) and substitute it back into (51). A straightforward computation finally yields,

$$\begin{aligned} r'^2 &= \frac{(1 - r^2)(r^2 - r_{min}^2)(r_{max}^2 - r^2)}{(1 - v^2)^2} \\ \mathbf{s}' &= -\frac{\omega_1 v}{(1 - v^2)} \end{aligned} \quad (52)$$

where, the roots could be formally expressed as,

$$r_{max,min} = \frac{1}{\sqrt{2\kappa}}(\omega_1 - 1 + \kappa^2(\mathfrak{h}_c - \omega_2\varrho_c^2))^{1/2} \left( 1 \pm \sqrt{1 - \frac{4\kappa^2(\omega_2\varrho_c^2 + \omega_1 - \mathfrak{h}_c)}{(\omega_1 - 1 + \kappa^2(\mathfrak{h}_c - \omega_2\varrho_c^2))^2}} \right)^{1/2} \quad (53)$$

subjected to the fact that we have set,  $\mathfrak{h}_c = \mathfrak{h}(\varrho = \varrho_c)$ . With the above solutions in hand, we now proceed towards evaluating all the conserved quantities in (48). We first compute the energy which turns out to be,

$$E = 2T\mathfrak{h}_c \int_{\epsilon_{IR}}^{r_{max}} \frac{dr}{r\sqrt{1-r^2}} \frac{1}{\sqrt{r_{max}^2 - r^2}} \quad (54)$$

where, like in the previous example, we set the lower limit ( $r_{min}$ ) equal to some IR cutoff,  $\epsilon_{IR}$  which is sufficiently small compared to unity namely,  $|\epsilon_{IR}| \ll 1$ . At this point, the reader should take a note on the fact that this is indeed a valid limit when one of the conserved charges ( $J_1 \rightarrow \infty$ ) goes to infinity [57]. On the other hand, for the sake of convenience we set the upper limit,  $r_{max} = 1$ . In the following, we would first like to explore the consequences of setting this upper limit equal to unity. Using, (53) one could try to have an estimate on the deformation parameter ( $\kappa$ ) for the theory and this simply implies,

$$\kappa^2 \approx \frac{(1 - \omega_1)^2}{(\omega_1 + 2)(2\omega_1 - 1)} \quad (55)$$

subjected to the choice,  $\omega_2\varrho_c^2 = 1$  and,  $\mathfrak{h}_c = -\omega_1$  which will be considered throughout the subsequent analysis. If we now demand,  $0 < \kappa^2 < 1$  which in turn implies that we push ourselves towards the limit without any deformation [49], then from (55) it immediately follows,

$$\frac{1}{2} < \omega_1 \leq 1 \quad (56)$$

which thereby non trivially constraints one of the important parameters of the theory which will eventually show up in the final dispersion relation that we are after. Furthermore, (56) also confirms that,  $(1 - \omega_1^2) \geq 0$  which is also quite consistent with the earlier observations made in connection to that of the three spin systems [49].

Considering these facts and evaluating the integral (54) we find,

$$E = 2T\mathfrak{h}_c \log \left( \frac{r}{\sqrt{1-r^2}} \right) \Big|_{\epsilon_{IR}}^1. \quad (57)$$

Next, we compute the angular momentum,

$$\begin{aligned} J_1 &= -2T\omega_1 \int_{\epsilon_{IR}}^1 \frac{dr}{r(1 + \kappa^2 r^2)} \\ &= -2T\omega_1 \log \left( \frac{r}{\sqrt{1 + \kappa^2 r^2}} \right) \\ &= -2\omega_1 T \log r \Big|_{\epsilon_{IR}}^1 + \kappa^2 \omega_1 T + \mathcal{O}(\kappa^3). \end{aligned} \quad (58)$$

Using, (57) and (59) we finally note the difference,

$$E - J_1 = -\frac{\sqrt{\lambda}\kappa^2\omega_1}{2\pi}\sqrt{1+\kappa^2} + \mathcal{A}^{(div)} \quad (59)$$

where, we separate out the divergent piece as,

$$\mathcal{A}^{(div)} = (\mathfrak{h}_c + \omega_1) \log r|_{\epsilon_{IR}}^1 - \frac{\mathfrak{h}_c}{2} \log(1-r^2)|_{\epsilon_{IR}}^1. \quad (60)$$

Therefore, like in the previous example corresponding to two spin magnon systems, we note that both  $E$  and  $J_1$  as well as their difference corresponding to the three spin configuration diverges as well. From (60), we note that the difference,  $E - J_1$  possesses both UV as well as IR divergences. However, by setting,  $\mathfrak{h}_c = -\omega_1$  one could in principle get rid off the IR divergences which thereby precisely falls in agreement with the earlier observations in the context of three spin giant magnon configurations [49].

We now go for computing the following conserved quantity,

$$\Psi = -2T \int_{\epsilon_{IR}}^1 \frac{dr}{r(1-r^2)} \quad (61)$$

where, we set,  $\varrho_c^2\omega_2 = 1$ . Clearly, the above integral (61) is divergent both at UV as well as near the IR limit. For the moment, we may forget about the IR divergence and try to regularize the divergence near the UV limit ( $r \rightarrow 1$ ). In order to do that, we set an UV cutoff,  $|\Lambda| \lesssim 1$  [49]. We also define a new variable,

$$R = 1 - r \quad (62)$$

such that,  $|R| \ll 1$  near the UV limit. Finally, re-expressing (61) in terms of this new variable (62) we obtain,

$$\Psi \approx T (1 + \log(|1 - \Lambda|)). \quad (63)$$

Next, we compute the time difference ( $\Delta t$ ) between the two endpoints of the open string which finally yields<sup>1</sup>,

$$\cos \frac{\Delta \tilde{t}}{2} \approx 1 + \cos \mathcal{D}(\Lambda) \quad (64)$$

where,

$$\mathcal{D}(\Lambda) \approx \log |1 - \Lambda|^{1/2} \quad (65)$$

is a UV divergent entity and is indeterminate in general as the argument in cosine becomes large in the limit,  $\Lambda \rightarrow 1$ .

Before we proceed further, it is noteworthy to mention that in the above derivation (64), we have implicitly assumed,  $v > 0$ . If we now further constraints our parameter space such that,  $\omega_1^2 \leq 1 - v^2$  and,  $0 < v < 1$  then combining these constraints with our previous arguments we finally land up into the physical parameter space for our theory,

$$1 - \omega_1^2 - v^2 \geq 0, \quad 1 - \omega_1^2 \geq v^2 \geq 0 \quad (66)$$

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<sup>1</sup>Here, we rescale the original angle,  $\Delta t \rightarrow \Delta \tilde{t} = \frac{\Delta t}{v}$ .

which thereby precisely matches to that with the parameter bound of the theory in order to be consistent with the three spin giant magnon solution [49].

Next, we compute the remaining conserved quantity,

$$J_2 = -2T \int_{\epsilon_{IR}}^1 \frac{r dr}{(1 - r^2)}. \quad (67)$$

Clearly, (67) is IR finite. However, it possesses UV divergences which we need to regularize. To do that, we follow our earlier prescription of UV regularization which finally yields,

$$J_2 \approx -T(1 - \log |1 - \Lambda|). \quad (68)$$

Finally, we compute the angle difference between the two end points of the open string,

$$\sin \frac{\Delta \tilde{\varphi}}{2} = -1 + \sin \mathcal{D}(\Lambda). \quad (69)$$

Following the original prescriptions [49], we now subtract all the UV divergent pieces and deal only with regularized entities which finally lead towards the following identity,

$$\begin{aligned} (E - J_1)_{reg} + \left( \sqrt{\Psi^2 + \frac{\hat{\lambda}}{4\pi^2} \cos^2 \frac{\Delta \tilde{\mathbf{t}}}{2}} \right)_{reg} - \left( \sqrt{J_2^2 + \frac{\hat{\lambda}}{4\pi^2} \sin^2 \frac{\Delta \tilde{\varphi}}{2}} \right)_{reg} \\ = -\frac{\sqrt{\lambda} \kappa^2 \omega_1}{2\pi} (1 + \kappa^2)^{1/2} \end{aligned} \quad (70)$$

which is the three spin giant magnon dispersion relation in  $\kappa$ - deformed  $AdS_3 \times S^3$  in the limit of large t'Hooft ( $\lambda \gg 1$ ) coupling. Eq.(70) essentially describes the superposition of  $J_2$  magnon bound states moving with momentum  $\Delta \tilde{\varphi}$  to that with another bound state of  $\Psi$  magnons moving with momentum,  $\pi + \Delta \tilde{\mathbf{t}}$  over the  $\kappa$ - deformed background [49]. Clearly, the newly obtained dispersion relation (70) is different from that of the earlier scenario without any  $\kappa$ - deformations [49]. In the present example, we have an additional nontrivial contribution on the R.H.S. of (70) that emerges solely due to presence of the background ( $\kappa$ ) deformations. Like in the previous example corresponding to two spin magnons, the above relation (70) might also have some deeper consequences from the point of view of the dual gauge theory and therefore this needs further attention.

## 5 Summary and final remarks

In this paper, we have made an attempt in order to understand the duality between super string theories formulated on deformed backgrounds to that with its gauge theory counterparts. In order to address this issue, in our analysis, we choose a specific path where using the conformal gauge conditions for the Polyakov action and considering the large value of the t'Hooft coupling ( $\lambda \gg 1$ ), we explore the two spin and three spin giant magnon configurations over  $\kappa$ - deformed  $AdS_3 \times S^3$  background [51]. It turns out that the corresponding dispersion relations receive non trivial contributions due to the presence of the background deformations which smoothly vanishes in the zero limit of the deformation parameter ( $\kappa$ ). The corresponding dual gauge theory interpretation associated with

these deformations is not clear at this moment and which is therefore worthy of further investigations.

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